QuickSort

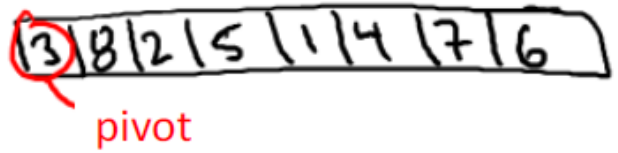
The Partition Subroutine

Design and Analysis of Algorithms I

Partitioning Around a Pivot

Key Idea : partition array around a pivot element.

-Pick element of array



-Rearrange array so that

-Left of pivot => less than pivot

-Right of pivot => greater than pivot

Note : puts pivot in its “rightful position”.

Two Cool Facts About Partition

1. Linear O(n) time, no extra memory
   [see next video]

2. Reduces problem size

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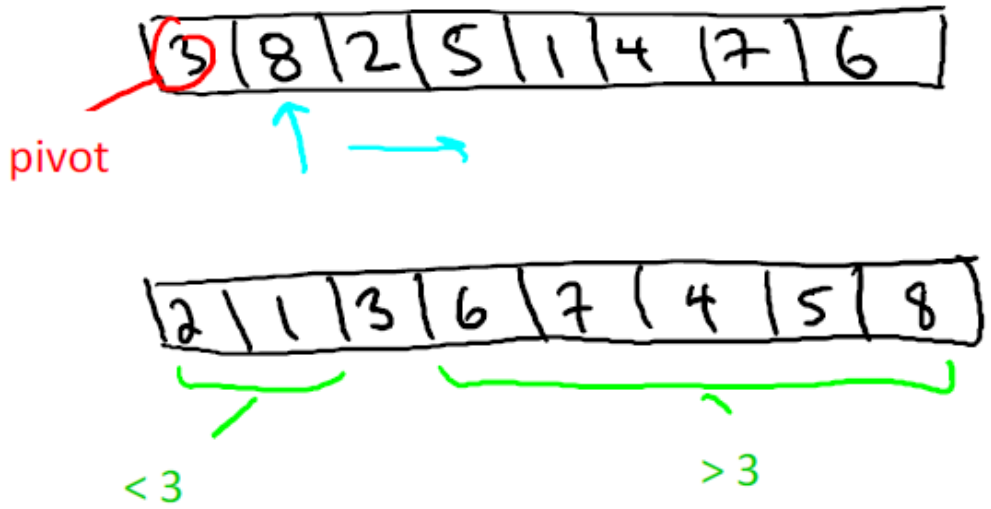
The Easy Way Out

Note : Using O(n) extra memory, easy to partition around pivot in O(n) time.

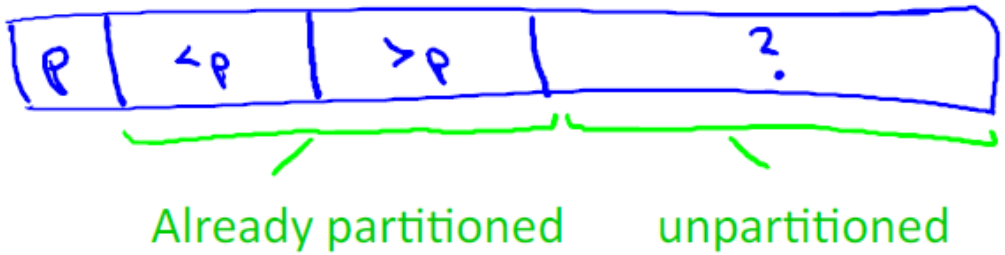


In-Place Implementation

Assume : pivot = 1st element of array

[if not, swap pivot <--> 1st element as preprocessing step]

High – Level Idea :



-Single scan through array

- invariant : everything looked at so far is partitioned

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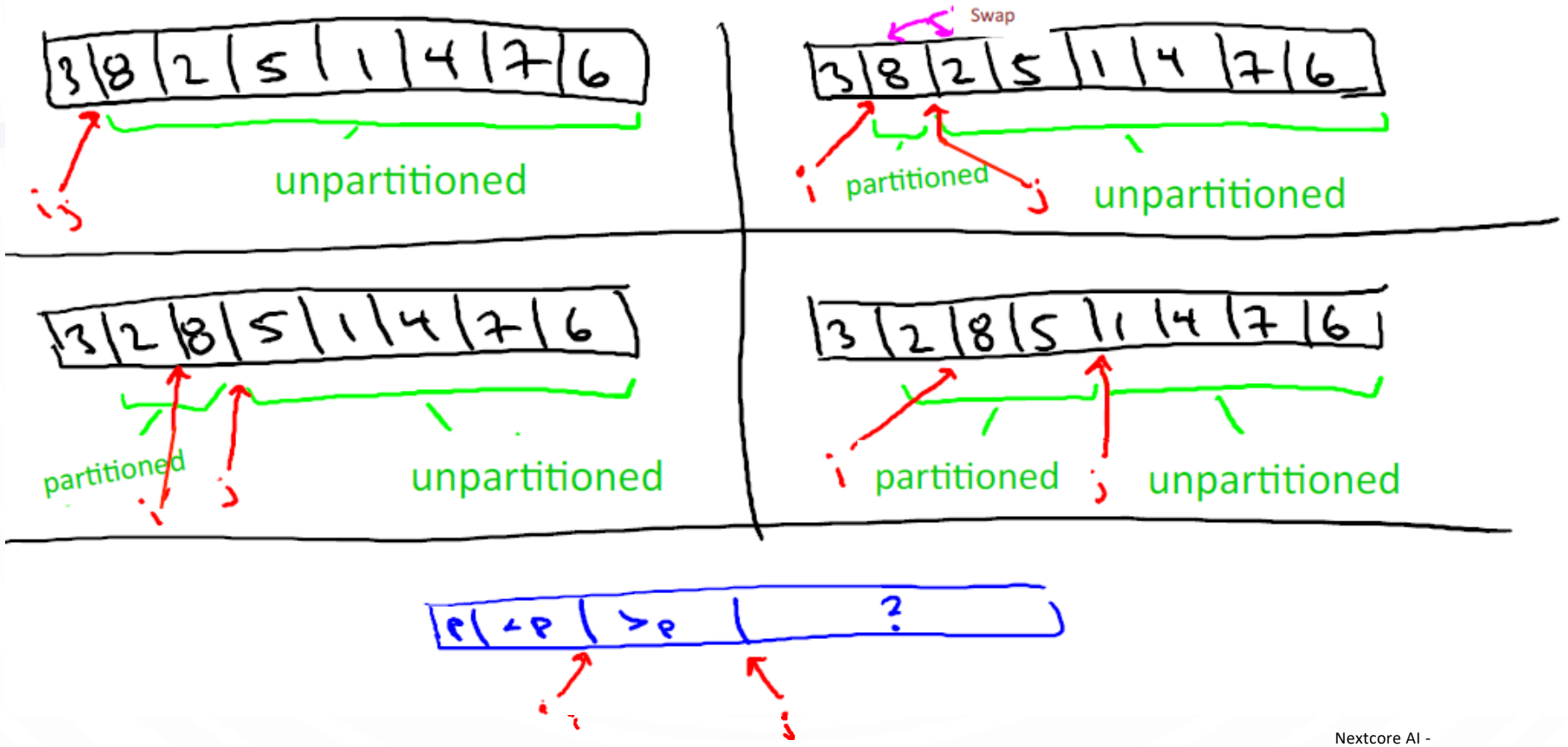
Partition Example



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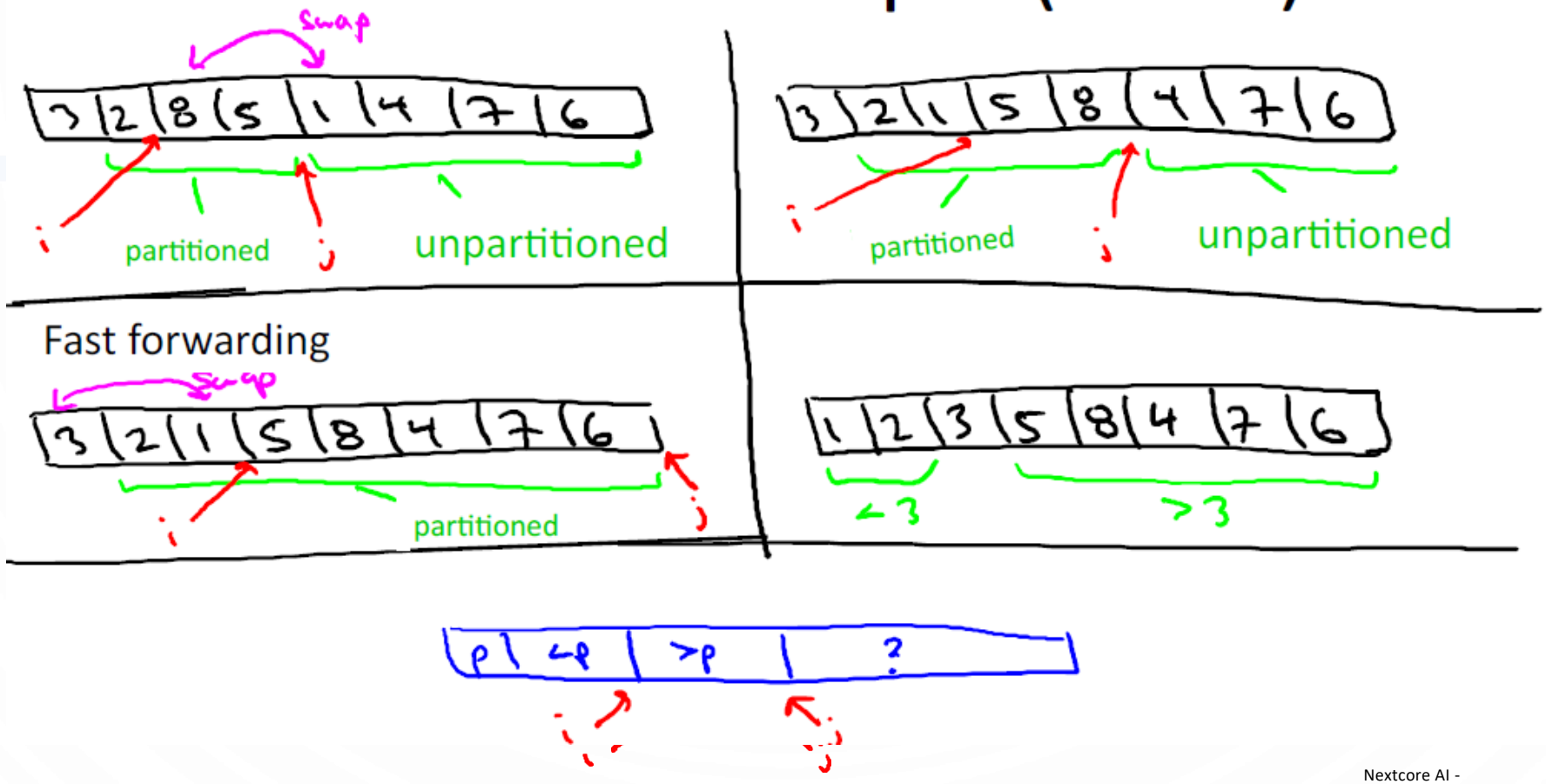
Partition Example (con'd)



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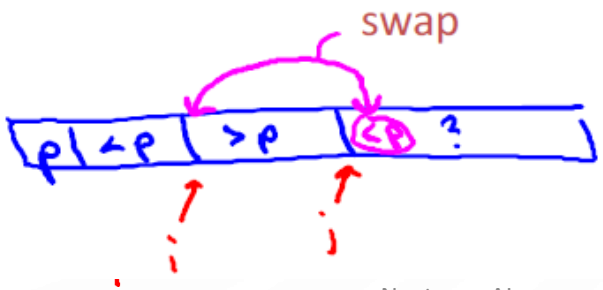
Pseudocode for Partition

```
Partition (A,l,r)                    [ input corresponds to A[l...r]]
        - p:= A[l]
        - i:= l+1
        - for j=l+1 to r
                - if A[j] < p                [if A[j] > p, do nothing ]
                    -swap A[j] and A[i]
                    - i:= i+1
        - swap A[l] and A[i-1]
```



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Running Time

Running time = O(n), where n = r – l + 1 is the length of the input (sub) array.

Reason : O(1) work per array entry.

Also : clearly works in place (repeated swaps)

Correctness

Claim : the for loop maintains the invariants :

1. A[l+1],..,A[i-1] are all
   less than the pivot
2. A[i],…,A[j-1] are all greater than pivot.

[Exercise : check this, by induction.]

Consequence : at end of for loop, have:

=> after final swap, array partitioned
around pivot.

Q.E.D

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