NEXTCORE AI

# Divide and Conquer

# Counting Inversions I

Design and Analysis of Algorithms I

# THE PROBLEM

Input : array A containing the numbers 1,2,3,..,n in some arbitrary order

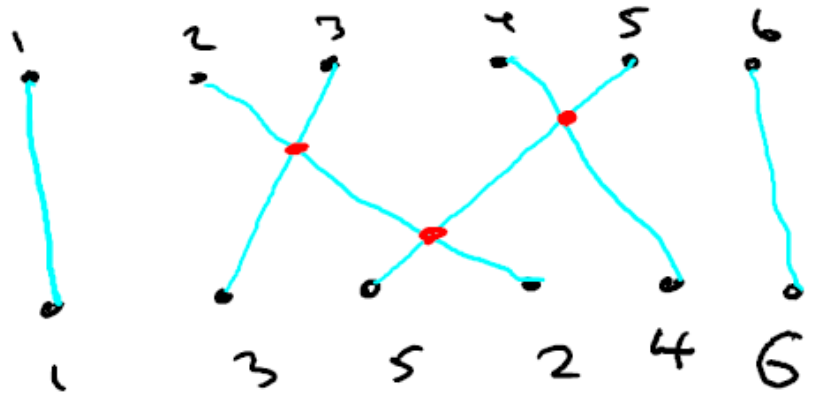
Output : number of inversions = number of pairs (i,j) of array indices with i<j and A[i] > A[j]

# Examples and Motivation

Example 


Inversions :

(3,2), (5,2), (5,4)




Motivation : numerical similarity measure between two ranked lists eg: for collaborative filtering

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What is the largest possible number of inversions that a 6-element array can have?

- 15
- 21
- 36
- 64

In general, $\binom{n}{2} = n(n-1)/2$

# High-Level Approach

Brute-force : $\theta(n^2)$ time

Can we do better ? Yes!

KEY IDEA # 1 : Divide + Conquer

Call an inversion (i,j) [with i<j]

- Left : if i,j < n/2
- Right : if i,j > n/2
- Split : if i<=n/2 < j

Note : can compute these recursively

need separate subroutine for these

# High-Level Algorithm

```
Count (array A, length n)
        if n=1, return 0
        else
                X = Count (1st half of A, n/2)
                Y = Count (2nd half of A, n/2_
                Z = CountSplitInv(A,n)
        return   x+y+z
```

CURRENTLY UNIMPLEMENTED

Goal : implement CountSplitInv in linear (O(n)) time then count will run in O(nlog(n)) time [just like Merge Sort]